

Beyond the Periphery: Child and Adult Understanding of World Map Continuity

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Abstract

It is well established that map projections make it difficult for a map reader to correctly interpret angles, distances, and areas from a world map. A single map projection cannot ensure that all the intuitive features of Euclidean geometry, such as angles, relative distances, and relative areas, are the same on the map and in reality. This article adds an additional difficulty by demonstrating a clear pattern of naiveté regarding the site at which a route that crosses the edge of a world map reappears. The argument is that this naïve understanding of the peripheral continuation is linear, meaning that the proposed continuation is along the straight line that continues tangentially to the original route when it crosses the edge. In general, this understanding leads to an incorrect interpretation concerning the continuation of world maps. It is only in special cases—such as radial routes on a planar projection and peripherally latitudinal routes on a cylindrical or pseudocylindrical projection with a normal aspect—that the actual peripheral continuation of the world map is linear. The data used in this article are based on questionnaires administered to 670 children aged 9–15 and 82 adults. This naïve understanding of the peripheral continuation, which leads to errors, was found to be entirely dominant among the children, regardless of the projection, and was clearly observed among the adults when the projection was cylindrical with a normal aspect.

Key Words: cartography, map projections, peripheral continuity, world maps.

The flattening of the globe into world maps has been a source of interest, confusion, and debate for two thousand years (Snyder 1993). A reliance on Euclidean geometry works perfectly well when reading maps of small areas—where angles, straight lines, relative distances, and relative areas closely approximate the real world. In Euclidean geometry, parallel lines never meet, the angular sum of triangles is always 180° , and the shortest path between points is a straight line, but on the surface of the Earth, the situation is different. The longitudes, for example, are parallel when they cross the equator but meet both at the North Pole and the South Pole, the angular sum of a spherical triangle varies, and the shortest path between points falls not on a straight line but along a great circle¹. To make it possible for a map reader to correctly understand a map of the world, a Euclidean-based understanding (that we are familiar with from our large-scale everyday life) has been the model for the qualities represented in map projections. Because not all of these qualities (such as conformality, the representation of great circles as straight lines, equidistance, and equal area) can be combined in the same projection of the entire world, it is necessary to choose or compromise among these qualities when a world map is drawn (Raisz 1938, 81–82; Fisher & Miller 1944, 27–28; Snyder 1987, 4). In this article, we argue that there is also a need to analyze the effect of different map projections on map readers' abilities to understand peripheral continuity, that is, to predict where a route that crosses the edge of a world map will reappear. For example, we show in this article that people (both children and adults) often mistakenly think that an airplane passing the southern edge of a rectangular world map continue the route from the northern edge (Figure 1). This misinterpretation, the belief that the North Pole (the most remote location of the world from the South Pole) is where you arrive when reaching the South Pole, is clearly problematic for the understanding of our world.

Distortion and Continuity

A characteristic of a map projection that does not accurately portray a certain criterion is often called a distortion and can be visualized by projecting familiar shapes using the map projection (Mulcahy & Clarke 2001, 168). Map projections that are not conformal have angular distortion, and map projections that are not equal-area have areal distortion. Perhaps the most well-known debate regarding angular and areal distortion was initiated by Peters (1983). He criticized the use of the conformal and cylindrical Mercator projection and attacked what he calls “the outdated theory” (Peters 1983, 67–68), in which conformality (or fidelity of angle) is the first of the ten “myths” of the outdated cartographic theory that he describes. He instead provided ten “attainable map qualities” and presented a cylindrical equal-area projection with a normal aspect² that fulfills them all. His projection gained considerable publicity and support among noncartographers, but many cartographers severely criticized the Peters projection (Monmonier 2004, 145–171). Three of Peters’ “attainable map qualities” require a cylindrical projection (Peters 1983, 105–118), and a response from seven major professional organizations of geographers and cartographers was to recommend the avoidance of all rectangular world maps (world maps based on a cylindrical projection with a normal aspect) because “[s]uch maps promote serious, erroneous conceptions by severely distorting large sections of the world by showing the round earth as having straight edges and sharp corners, by representing most distances and direct routes incorrectly, and by portraying the circular coordinate system as a squared grid” (The American Cartographer 1989, 222). This recommendation advises against both the Peters projection and the Mercator projection and instead promotes pseudocylindrical projections, such as the equal-area Mollweide projection (see Figure 6B), and interrupted projections, such as the equal-area Goode homolosine projection (see Figure 2C).

The distortion of a map varies over its surface; by selecting and combining the different parts of the world from different projections, it is possible to reduce the distortion in an interrupted world map. Map projections with two or more central meridians³ have been used since the sixteenth century (Figure 2A). Additionally, in the latter half of the nineteenth century, asymmetrically interrupted projections began to be developed to preserve the qualities and the continuity of more complex areas of maps (Dahlberg 1962, 50). For example, the Goode homolosine projection, which was invented in 1923, is an equal-area projection that has reduced angular distortion because of these interruptions and can preserve the continuity of oceans by interrupting the land masses (Figure 2B) or the other way around (Figure 2C), depending on whether the ocean or land is of interest (Goode 1925; Snyder 1993, 279). Spilhaus and Snyder (1991), by contrast, proposed interrupting the map along the shorelines to preserve the continuity of both land and oceans (Figure 2D).

All world maps are interrupted to some degree and therefore have a periphery. Some cartographers have tried to reduce the discontinuity of the periphery. In 1879, Peirce published the quincuncial projection, which reduces the edge of the world map to points when the map is drawn periodically (Figure 3). Peirce suggested that the projection could be used for meteorological and magnetological purposes for which “it is convenient to have a projection of the sphere which shall show the connection of all parts of the surface” (Peirce 1879, 394). The National Geographic Society has presented another method of removing the discontinuity by producing an interactive map with two rotatable hemispheres. The map was constructed in this manner to make it possible to measure the distance across the equator, and any chosen point on the periphery can be made continuous by rotating the hemispheres (Chamberlin 1947, 71).

The focus has previously been on where to avoid interrupting the surface to preserve the continuity of the areas of interest. However, it is also important to consider the

consequences that such interruptions have on the ability of map readers to grasp the continuation of the interrupted areas. Because all map projections have a periphery, an understanding of where a path on a map continues when the map edge is crossed is relevant for all world maps.

In this article, we analyze how map readers understand the peripheral continuity of world maps using a sample of 670 children and 82 adults. We attempt to determine whether there is a naïve understanding of the peripheral continuity of world maps that differs from the actual peripheral continuity. In this study, naïve understanding means that the understanding or reasoning to solve a task is not sufficiently developed to reach a correct solution; the term applies only to the specific task and implies nothing about the respondent in general.

The hypothesis in this article is that the peripheral continuity is perceived as linear according to a naïve understanding of map projections. This hypothesis is based on the assumption that the naïve understanding of the map reader is built on a comprehension based on Euclidean geometry.

In this study, there are limitations in the applied psychological perspectives when studying how map readers understand the peripheral continuity of world maps. The main focus in this study is to model how map readers generally respond to the periphery of world maps; individual factors are not studied.

Hypothesis: Linear Peripheral Continuity

In Euclidean geometry, the shortest path between any two points on a plane is the straight line segment connecting them. If we inaccurately apply this theorem to a world map, where non-Euclidean spherical geometry should be used, and combine this with the understanding that if we continue in the same direction to complete a lap around the world, we will come back to the same place from the other direction, then, we will acquire the naïve idea that the peripheral continuation of the route is along the straight line that continues

tangentially to the original route where it crosses the edge. This type of peripheral continuity based on the Euclidean spatial conception is in this article called *linear peripheral continuity* (Figure 4). If the Euclidean-based understanding of great circles is followed, the peripheral continuity would always be perceived as linear. Because the surface of the round world is non-Euclidean and great circles are not always presented as straight lines on a world map, this perception does not always correspond to the actual case (e.g., Figure 4B and 4C). However, in some cases, great circles are actually presented as straight lines on world maps. In planar projections, for example, all straight lines crossing through the center of the projection are great circles (Snyder 1987, 141). This quality gives planar projections an actual radially linear peripheral continuity. Additionally, many projections portray the equator (the only latitude that is a great circle) as a straight line. This quality produces an actual linear peripheral continuity along the equator (e.g., Figure 4A). Actual linear peripheral continuity is in fact always achieved parallel to the secants or tangent⁴ for cylindrical and pseudocylindrical projections. For cylindrical projections with a normal aspect, the lines parallel to the secants or tangent are latitudes, and this effect can therefore be called actual latitudinally linear peripheral continuity. However, when projected according to a cylindrical or pseudocylindrical projection with a normal aspect, peripherally latitudinally great circles other than the equator are not presented as straight lines. Instead, the actual latitudinally linear peripheral continuity results from the great circle to whose tangent a latitude when passing the edge returns at the same latitude after a complete lap around the world (Figure 5). It is this actual latitudinally linear peripheral continuity that makes it possible to assemble multiple copies of a cylindrical projection side by side.

For cylindrical projections with a normal aspect, there is no longitudinally linear peripheral continuity (Figure 4C). However, in many video games, a method similar to a

combination of latitudinally and longitudinally linear peripheral continuity has been used since the 1960s game *Spacewar!* to make objects that cross the edge return to the screen.

Cognitive Studies

The field of academic cartography in the United States started in the beginning of the twentieth century with a few but important scholars, such as Goode, Raisz, Smith, and Harrison. However, in the period following World War II, there was a great expansion of academic cartography in many graduate education centers around the United States (McMaster & McMaster 2002, 306–309). During the second half of the century, there was also an intensification of empirical studies in cognitive map design research to improve cartographical problem solving and communication (Montello 2002). Most empirical map design research has studied thematic mapping symbols—a part of map design that is easier than reference maps to vary in an experimental setting and provides clear information that the map reader is supposed to recognize (Montello 2002, 285–286). For reference maps, a type of information that the map reader is supposed to obtain clearly is that concerning geographical relations, such as distance and areal relations. For world maps, those geographical relations are related to the cognitive aspects of map projections. Previous empirical studies of the cognitive characteristics of world map projections include that of Battersby and Montello (2009), who concluded that the perceived areal relations of landmasses are shaped by the manner in which people naïvely understand areas of world maps in combination with the projections used in society. Regarding people’s abilities to understand distances on world maps, Carbon (2010) concluded from his study that persons without personal experience of the Earth as a sphere tend to see the world as flat when they attempt to estimate distances on a world map.

Children sometimes have a conflicting understanding of the world as round and, at the same time, as the “flat” ground they are living on. This phenomenon has been thoroughly

studied in the Piagetian tradition by Nussbaum and colleagues and by Vosniadou and colleagues (Ehrlén 2007, 17). Vosniadou and Brewer (1992), for example, showed in their study of 1st-, 3rd-, and 5th-grade children that different mental models of the Earth could be identified (for example, the rectangular earth, the disc earth, the dual earth, the hollow sphere, and the flattened sphere). Piaget and Inhelder (1956) presented topological space, projective space, and Euclidean space as the three stages of spatial understanding. Downs and Liben (1991) argued that understanding the fundamentals of map projections assumes a Euclidean understanding, which has not developed in all college students. Note that the Piagetian idea about stages of spatial understanding has been questioned by many scholars. Specifically, the following concerns have been raised: at what ages do the different stages occur, can the steps be clearly separated, and is the understanding inherent or is it developed via interactions between people and the environment (Blaut 1997, 172).

Almost a century ago, psychologists began attempts to differentiate spatial intelligence factors from general intelligence (Harris, Hirsh-Pasek & Newcombe 2013, 110). The abilities of mental rotation and mental (paper) folding are two factors of spatial intelligence that have been extensively studied. Mental folding concerns understanding a flat representation of a three-dimensional object. Psychologists have also studied the peripheral continuation of geometric shapes by asking respondents to identify the edges that will meet when an unfolded cube is folded together (Shepard & Feng 1972). Note that the peripheral continuation of certain interrupted map projections can be understood by mental folding. Mental bending (a non-rigid mental transformation similar to mental folding but without fold lines) has also been studied (Atit, Shipley & Tikoff 2013), and the continuation of cylindrical map projections can be understood to a certain extent through this mental exercise. Mental folding and mental bending might therefore be important abilities for understanding the peripheral continuation of world maps. However, to understand the continuation of many of

the common world map projections, a more demanding mental exercise is needed in which, for example, points on the round world can be represented as lines or circles on a world map.

Methods and Data

The hypothesis of linear peripheral continuity was tested using a questionnaire in which respondents were asked to mark the point at which they thought a route crossing the edge of a world map would continue. The questionnaire was in Swedish but is described here in English. The title of the questionnaire, *The Earth is round, the map is flat*, was followed by the instructions: *Airplanes A, B, and C pass the edge of the world map without turning. Draw three arrows on the map below to show where and in what direction the three different airplanes will return to the map when the edge of the map has been passed. Mark the drawn arrows with the letters A, B, and C.*

Below the instructions was an image of one of five world maps (Figure 6). Five different map projections were used, but each respondent was given only one projection. The five world map projections used in this study included a cylindrical, a pseudocylindrical, a planar, and two different interrupted projections. This selection made it possible to study the understanding of peripheral continuation in cases of actual radially and latitudinally linear peripheral continuity alongside cases without actual linear peripheral continuity. The selection also made it possible to compare cases of different complexity regarding mental folding. Because Miller's projection from 1942 is cylindrical, it has an actual latitudinally linear peripheral continuity (Figure 6A). This projection is a compromise between the Mercator and other cylindrical projections (Snyder 1987, 86). The Mercator projection is a frequently used cylindrical projection, which has recently been utilized even more due to its application in web maps. The reason for choosing the Miller projection instead of the Mercator projection is because the former resembles the Mercator projection but, unlike the Mercator, accommodates the entire surface of the world (Miller 1942, 430), a desirable

quality when focusing on peripheral continuity. The path of continuation for planes A and C on the map based on the Miller projection is possible to determine using mental bending. Because Mollweide's equal-area map projection from 1805 is pseudocylindrical, it has an actual latitudinally linear peripheral continuity (Figure 6B). The Berghaus star projection from 1879 is known from the logo of the Association of American Geographers (Figure 6C). This projection has no actual linear peripheral continuity; however, the projection is interrupted in such a manner that the peripheral continuation can be determined with mental folding. Fuller's interrupted map projection from the 1940s has no actual linear peripheral continuity (Figure 6D). This projection is a planar arrangement of a polyhedron (Dahlberg 1962); therefore, the peripheral continuation is possible to determine using mental folding. Lambert's azimuthal equal-area projection from 1772 is a planar projection and has actual radially linear peripheral continuity (Figure 6E). The five different world maps were provided with a graticule that according to Dahlberg (1961, 213), "provides the chief visual basis for relating the map to the globe". The omission of a graticule is a noted problem for interrupted projections because the interruptions can then be understood as merely more ocean (Snyder 1993, 198).

Beneath the map, the following instructions were given: *How certain are you that the arrows you have drawn are drawn at the right location and in the right direction? Place an X on the three lines below to show how certain you are.* The questionnaire then included three lines marked with the labels *Arrow A*, *Arrow B*, and *Arrow C*. The lines ranged from *complete guess to completely certain*.

The answers were registered using a transparent raster and quantified to measure the closeness of each answer, both to the actual continuation and to the linear peripheral continuity. Four of the five measurements calculated for each proposed continuation are presented in Figure 7. Two measurements for the distance between the proposed and actual

continuation were used. Distance d measures the geographical distance and considers the localization of actual continuation to exist at multiple points on the map (if the airplane is allowed to turn when the edge is crossed), whereas distance m measures the distance on the map and considers only one of those points on the map to be correct (if the airplane does not turn when the edge is crossed). For airplane B on the world map using the Mollweide projection, angle α (angle on the map related to the tangent of the original route when crossing the edge) is especially important in determining whether the proposed continuation is linear because measurements d , m , and s are insufficient. The reason for this insufficiency is that the actual continuation is in this case located along the tangent of the original route in this case; however, the continuation has a different direction. The fifth measurement c is the self-rated certainty of the respondents answer, ranging from 0 percent (complete guess) to 100 percent (completely certain).

When statistically testing the resulting accumulated frequencies of the measurements, a comparison was made with the theoretical case in which every point along the edge has the same probability of being a proposed continuation. Those expected frequencies, indicating how often a random location along the edge is counted as actual peripheral continuity or linear peripheral continuity, differ for different cases. The case is the combination of the location and direction for the original route when crossing the edge, together with the projection. The expected frequencies for all the projections and airplane routes used in this study can be found in the bottom section of the table in the Appendix and is visualized as black horizontal bars in the diagrams in Figure 7, 8, 9 and 10. Those expected frequencies acted as the null hypothesis, H_0 , when testing whether the respondents tend to answer correctly and whether they tend to answer according to the hypothesis of linear peripheral continuity. A binomial distribution test was used to calculate p-values because we tested a null hypothesis using frequencies of dichotomous data (Welkowitz, Cohen & Lea 2011, 438).

The questionnaires were given in Sweden during 2013. The children were in the 3rd, 5th, 7th, and 8th grades in elementary school in the town of Arvika and were 9–15 years old. The questionnaires were administered to the children with the assistance of the teachers, and the children had no briefing on cartography or projections before they answered the questions. The adult group consisted of a mixture of people, and a majority had basic training in cartography, including a lecture on map projections, at the university level. The adults completed a stand-alone test with only the questions for this study, whereas the children completed a questionnaire that included additional geography-related questions outside the scope of this article.

Results

The children who were given the questionnaire included 343 boys, 326 girls, and 1 child whose sex was not indicated. The adults included 39 men, 41 women, and 2 individuals who did not want to state their sex. In total, 752 persons were given the questionnaire, and 461 answered with at least one proposed continuation, which produced a total response rate (RR) of 61 percent (more details are provided in Table 1).

Figures 8, 9, and 10 present statistics for the measurements d , m and s for the proposed continuations. Details for all measurements can be found in the Appendix. The statistics were calculated separately for the children and adults and for each airplane on each map projection. The accumulated frequencies of the measurements relative to the actual respective linear continuation are presented in the figures, together with an indication of whether the frequencies are significantly higher than what would be expected with randomly distributed proposed continuations along the periphery. The number of answers n for the measurement α varies in certain cases because not every respondent indicated the direction of the proposed continuation. In Figures 7, 8, 9, and 10 the proposed continuations without directions are marked as a circle instead of an arrow. The width of the arrows and circles on

the maps is proportional (the smallest arrows are slightly thicker to be readable) to the number of answers proposed for that location and direction.

In all five cases of actual linear peripheral continuation, the continuations proposed by both the children and adults were frequently linear; therefore, these continuations were frequently close to the actual continuation (Figure 8). These results are all significantly higher (with a p-value less than 0.001) than what would be expected if the proposed continuations were randomly distributed along the edge of the map. The cases using actual latitudinally linear peripheral continuation showed the highest mean values of certainty for both the children and adults.

In the cases in which linear peripheral continuation did not apply, the proposed continuations by the children were significantly (with a p-value less than 0.01) more often linear than what would be expected with randomly distributed continuations along the edge (Figure 9). In the three cases in which linear peripheral continuation did not apply, the proposed continuations by the children were significantly more often close to the actual continuation than that for randomly distributed continuations (Figures 9H, 9I, and 9J).

Among the adults, a significant (with a p-value less than 0.05) tendency to propose a linear continuation in the cases without an actual linear continuation could be found only for the cylindrical Miller projection (Figure 10A and 10B, respectively).

The selection of the projections in this study was also made to compare the complexity of the cases regarding mental folding. However, the extremely strong trend among the children to answer according to the idea of linear peripheral continuation together with the inclusion of too few adults in the study made it impossible to detect significant patterns regarding this complexity.

Discussion

The results clearly verify the hypothesis that people answer according to the idea of linear peripheral continuity, even in those cases in which such continuity does not apply. This hypothesis was true for the children using all the map projections but only for the adults using the map based on the cylindrical projection with a normal aspect. The hypothesis was constructed assuming a spatial understanding of Euclidean geometry. If one follows the convention of using a Euclidean-based understanding (which works perfectly well for maps of small areas) as the model for the qualities of world map projections, then the desirable quality regarding peripheral continuation of world map projections would be *actual linear peripheral continuity*. A good choice of projection for achieving this quality would be a planar projection that accommodates the entire world (similar to the Lambert azimuthal equal-area projection) and that has the quality of actual radially linear peripheral continuity. By choosing a planar projection that shows the entire world, people who follow the idea of linear peripheral continuity will always find the actual continuation when crossing the edge perpendicularly.

We now consider theories from previous cognitive studies in this discussion of the results, starting with the study by Vosniadou and Brewer (1992), to determine whether different mental models of the Earth can be used for understanding our results. In the results, we can observe certain proposed continuations among the children that do not cross the edge but instead are located along the original route before it crossed the edge. Those answers can perhaps be at least partially explained by a mental model of the Earth as “the rectangular earth” or “the disc earth”. If we instead endeavor to explain why the hypothesis is true, it is tempting to try to connect the idea of linear peripheral continuity with the mental model of “the dual earth”, “according to which there are two earths: a round one that is located in the sky and a flat one where people live” (Vosniadou & Brewer 1992, 550). The evolution of this

mental model bears a resemblance to how the hypothesis of linear peripheral continuity was developed; namely, as a combination of an idea that works for a round earth—that if we continue a complete lap around the world in the same direction, we will come back to the same place from the other direction—and an idea that works for a flat earth—that the shortest path between any two points is the straight-line segment connecting them. However, there are reasons why “the dual earth” is not a good explanation for the idea of linear peripheral continuity. The first reason is that the mental model stipulates that the round earth is in the sky; therefore, its roundness should not affect the movement of objects on the flat earth where we live. The second reason is that the idea of linear peripheral continuation could also be found among adults who had basic training in cartography at the university level. These adults know perfectly well that we live on a round earth. The third reason is that the mental model of “the dual earth” should be less common in higher grades among the children, but this expectation was not observed. To discern the differences by grade within the group of children, we chose the answers for a case without actual linear continuation and divided the children by grade (Figure 11). Except for a continual increase in the response rate from 35 percent in grade 3 to 100 percent for the adults, no progression related to grade was observed among the children in the studied age span. A slightly increasing tendency to answer according to linear peripheral continuation was observed, which should have been a decreasing tendency if the linear tendency were connected to “the dual earth” concept. This increase in answering according to the idea of linear peripheral continuity may instead be related to the development of spatial conception in children, according to Piaget and Inhelder (1956). The Euclidean spatial concept that forms the foundation of the hypothesis of linear peripheral continuation for this article is, according to Piaget and Inhelder, a spatial concept that develops during childhood. The increased rate of children answering according to the concept of linear peripheral continuation as they age may be a result of reaching this level of

spatial conception. An important finding that arises in this context is that this maturation does not explain why the adults more frequently proposed a continuation close to the actual continuation, whereas the children tended to answer more linearly than the adults in the cases without actual linear peripheral continuation. These patterns indicate that in addition to the three stages of spatial understanding, including topological, projective, and Euclidean space, there is also a non-Euclidean level of spatial understanding that helped many of the adults to find the actual continuation. While Downs and Liben (1991) argued that understanding the fundamentals of map projections assumes a Euclidean understanding, we argue, based on our results, that a Euclidean understanding is not sufficient for understanding the peripheral continuity of map projections. A non-Euclidean understanding with a spatial conception based on spherical geometry is also crucial. Observe that a non-Euclidean understanding in this case is not the same as a formal mathematical understanding of non-Euclidean geometry. Instead, a non-Euclidean understanding is an ability to link together a flat representation of a spherical surface with a sphere. The naïve understanding of map projections based on a Euclidean understanding can be described as follows:

A child looking at a map of the world in Mercator projection cannot help believing that Greenland is larger than Australia; he simply *finds* it larger. The projection employed is not the usual principle of copying which we use in all visual comparisons or translations, and his training in the usual rule makes him unable to “see” by the new one. (Langer 1951, 79–80)

It is possible to work around this naïve understanding in a formal manner. For example, Olson (2006) provides a step-by-step tool to use for visually judging if a map projection is conformal, equal-area, or neither. However, the quotation by Langer continues: “It takes sophistication to ‘see’ the relative sizes of Greenland and Australia on a Mercator

map. Yet a mind educated to appreciate the projected image brings the eye's habit with it. After a while, we genuinely 'see' the things as we apprehend it." (Langer 1951, 80)

What Langer describes here is not about ticking boxes. Instead, what she describes is about genuinely seeing areal relations in a world map that is not based on an equal-area projection. Vinge (1951) argues, based on the text of Langer, that the Mercator projection has many merits and that "[i]n social science[,] the Mercator, instead of being an object of scorn, can be used for a wide range of purposes provided the peculiarities of its symbolism are understood by the students" (Vinge 1951, 33). By analogy, it is not necessary to choose a projection with actual linear peripheral continuity to make it possible for the map reader to determine the peripheral continuation of the world map, if the map reader can reach a non-Euclidean understanding instead.

This non-Euclidean understanding that helps a map reader to find the actual peripheral continuation (when linear peripheral continuity does not apply) can most likely be obtained in different ways. One way is presumably by personal experience of the Earth as a sphere, in line with the results from the study by Carbon (2010); however, map design can most likely also facilitate this process.

The results verified a tendency for the adults to incorrectly answer according to the idea of linear peripheral continuity only when the map was based on a cylindrical projection with a normal aspect. The straight lines and right angles of these projections most likely did not help the map reader to ignore the Euclidean understanding. A first map design step to facilitate the attainment of a non-Euclidean understanding is therefore to follow the recommendation (The American Cartographer 1989) to avoid all rectangular world maps. The next map-design-related idea to overcome the Euclidean understanding is based on the fact that only in three cases for the children in this study was the frequency of the proposed continuation significantly closer to the actual continuation than that for randomly distributed

continuations (except in the cases with actual linear peripheral continuity). Note in all three cases that the route crossed the edge at a recognizable shape. In one case, the recognizable shape was the point of the star (see Figure 9I). In another case, Australia was split into two (see Figure 9J). In the last case, the edge was crossed where the graticule closed together against the South Pole (see Figure 9H). This group of three cases indicates that pairs of recognizable (geographical or cosmetically) objects that mark continuation along the periphery can perhaps help the map reader to understand where the map continues. A third possible idea is based not on the empirical results from this study but on the way in which the hypothesis regarding linear peripheral continuity was developed. This idea is to complement the grid of longitudes and latitudes with a network of great circle arcs, in the manner proposed by Fisher and Miller (1944, 11). This could help a map reader to understand the great circles and to overcome the Euclidean understanding of the shortest paths between points on a world map. Those map-related ideas could be a starting point for further studies. Another approach for further studies is to determine how the ability to find the correct peripheral continuation corresponds to individual spatial abilities, such as the ability to accomplish mental folding.

This study was conducted using printed world maps. Are the results also relevant when web maps and digital globes are used? Many interactive web maps are based on the Mercator projection and using the actual latitudinally linear peripheral continuity to make the map continuous in the east-west direction. This design assists the map reader by connecting the western and eastern edges; however, it is unclear where you continue when passing across the northern or southern edges of the map. In the north-south direction, the reader can therefore still be misled by the concept of linear peripheral continuity. However, similar to physical globes, digital globes can probably assist the map reader in better understanding the roundness of Earth.

Conclusion

In this article, the naïve understanding of peripheral continuation is proven to be linear, meaning that the proposed continuation is along a straight line that is tangential to the original route when the route crosses the edge. This tendency to propose a linear continuation in the cases without actual linear peripheral continuation was found to be entirely dominant among the children, regardless of the projection. This tendency was also clearly visible among the adults (even though the majority had academic training in cartography that included a lecture on map projections) for a projection that was rectangular (cylindrical with a normal aspect). Downs and Liben (1991) emphasize that Euclidean understanding is necessary for learning map projections. Based on the results of the present article, we argue that Euclidean understanding is not sufficient to understand the peripheral continuation of the mapped world. A non-Euclidean understanding based on spherical geometry is also required. These results reveal a previously unknown problem regarding the understanding of world map projections, i.e., an understanding in which many children and adults make the mistake of thinking that they will end up at the North Pole if they pass the southern edge of a rectangular world map. This new knowledge should be useful for improving the design of world maps. One way to help a map reader to determine the actual peripheral continuation might be to provide guidance in the identification of the paths of great circles. This assistance could be provided, for example, by drawing a network of great-circle arcs on the world map, such as those proposed by Fisher and Miller (1944, 11), or by avoiding rectangular projections, which because of their straight lines and right angles, can give an impression that the movements between two points on the globe occur along a straight line on the map. In this manner, the results of the present study—in which the rectangular projection resulted in a tendency toward the naïve linear understanding, even among the adults—contribute to the recommendation to avoid rectangular projections when designing world maps (The American

Cartographer 1989). According to the results of the present study, another method for helping a map reader determine the correct continuation is to split recognizable shapes (preferably cosmetically but could also be geographical objects) when interrupting the world map. The Berghaus star projection split Australia in two, and this split likely served as a point of reference when the respondents of this study were mentally folding the star together to identify the peripheral continuation. Now that this naïve understanding has been elucidated, peripheral continuation of world maps can be addressed by researchers, educators, and mapmakers to improve our understanding of our world.

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Notes

1. A *great circle* is the intersection of the surface of a sphere and a plane that intersects the center of the sphere. The shortest path between two points on the surface of a sphere follows the great circle that passes through these two points.
2. A *cylindrical projection with a normal aspect* (or equatorial aspect) represents longitudes and latitudes as straight lines that form a grid with right angles. A world map based on such a projection is called a rectangular world map.
3. The *central meridian* is the longitude that defines the center of a projected coordinate system.
4. When constructing a cylindrical projection, the intersection of the surface of the spherical Earth and the undeveloped surface (in this case a cylinder) is defined by the *secants* or the *tangent*. For cylindrical projections with a normal aspect, the tangent is the equator, or the secants follow two latitudes. This tangent is not to be confused with the tangent of the original route.

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Appendix

Statistics for the proposed continuations in the 15 cases. Each of the five different projections includes three continuations represented by the arrows A, B, and C. The gray fields indicate the five cases with actual linear peripheral continuation, and bold font indicates the measurements of the linearity of the proposed continuations.

Projection	Miller cylindrical			Mollweide			Berghaus star			Fuller			Lambert azimuthal				
	Arrow	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	
Actual linear peripheral continuation	lat.			lat.									rad.	rad.	rad.		
Frequency of the continuations proposed by the children	n	79	79	79	84	86	87	60	59	60	71	72	71	75	77	75	
	n for α	73	72	72	81	80	83	56	55	56	68	69	68	68	69	68	
	$d \leq 1000$ km	75 ^{***}	7	4	82 ^{***}	10	9	19 ^{***}	4	9 ^{**}	2	2	7	75	77	75	
	$d \leq 2000$ km	78 ^{***}	7	7	84 ^{***}	10	10	27 ^{***}	4	17 ^{***}	3	2	7	75	77	75	
	$m \leq 1$ cm	76 ^{***}	0	3	81 ^{***}	9	5	9 ^{**}	5	16 ^{***}	2	6	12 ^{***}	67 ^{***}	68 ^{***}	71 ^{***}	
	$m \leq 2$ cm	76 ^{***}	2	3	81 ^{***}	10	6	9	5	16 [*]	7	7	14 ^{**}	72 ^{***}	74 ^{***}	73 ^{***}	
	$s \leq 1$ cm	78 ^{***}	67 ^{***}	50 ^{***}	84 ^{***}	50 ^{***}	56 ^{***}	27 ^{***}	39 ^{***}	25 ^{**}	36 ^{***}	24	38 ^{***}	69 ^{***}	58 ^{***}	56 ^{***}	
	$s \leq 2$ cm	78 ^{***}	70 ^{***}	70 ^{***}	84 ^{***}	71 ^{***}	76 ^{***}	40 ^{***}	45 ^{***}	28 ^{**}	41 ^{***}	54 ^{***}	51 ^{***}	73 ^{***}	72 ^{***}	73 ^{***}	
	$\alpha \leq 18^\circ$	71 ^{***}	63 ^{***}	63 ^{***}	76 ^{***}	66 ^{***}	57 ^{***}	45 ^{***}	36 ^{***}	25 ^{***}	40 ^{***}	48 ^{***}	45 ^{***}	60 ^{***}	61 ^{***}	62 ^{***}	
	\bar{c}	73	64	56	71	55	56	42	40	35	35	30	34	60	61	63	
St. dev. c	31	34	33	32	34	33	33	33	31	31	29	30	35	34	34		
Frequency of the continuations proposed by the adults	n	17	17	17	17	17	17	15	15	16	15	15	15	16	16	16	
	n for α	17	17	17	17	17	17	14	14	15	15	15	15	15	15	15	
	$d \leq 1000$ km	16 ^{***}	8	10 ^{***}	17 ^{***}	7 ^{**}	6 [*]	8 ^{***}	2	6 ^{***}	6 ^{***}	4 ^{**}	8 ^{***}	16	16	16	
	$d \leq 2000$ km	17 ^{***}	8	11 [*]	17 ^{***}	7	8 [*]	10 ^{***}	2	8 ^{***}	6 ^{***}	4 ^{**}	8 ^{***}	16	16	16	
	$m \leq 1$ cm	17 ^{***}	2	11 ^{***}	17 ^{***}	7 ^{***}	8 ^{***}	2	2	8 ^{***}	7 ^{***}	6 ^{***}	12 ^{***}	9 ^{***}	12 ^{***}	13 ^{***}	
	$m \leq 2$ cm	17 ^{***}	3	11 ^{***}	17 ^{***}	7 ^{**}	8 ^{**}	2	2	10 ^{***}	9 ^{***}	6 ^{**}	13 ^{***}	10 ^{***}	12 ^{***}	14 ^{***}	
	$s \leq 1$ cm	17 ^{***}	7 ^{***}	5 [*]	17 ^{***}	7 ^{**}	3	3	5	1	0	3	1	9 ^{***}	6 ^{***}	8 ^{***}	
	$s \leq 2$ cm	17 ^{***}	7 ^{**}	6	17 ^{***}	7	4	8	5	1	1	8	1	10 ^{***}	12 ^{***}	14 ^{***}	
	$\alpha \leq 18^\circ$	17 ^{***}	9 ^{***}	13 ^{***}	17 ^{***}	3	12 ^{***}	9 ^{***}	6 ^{**}	0	3	5 [*]	5 [*]	9 ^{***}	12 ^{***}	15 ^{***}	
	\bar{c}	78	42	54	88	57	55	33	27	30	37	42	50	41	44	43	
St. dev. c	26	29	20	21	26	27	28	24	31	27	31	30	32	32	31		
Expected frequency [%]	$d \leq 1000$ km	4	33	8	9	14	14	12	4	6	5	3	5	100	100	100	
	$d \leq 2000$ km	12	37	40	17	23	26	24	8	10	6	5	8	100	100	100	
	$m \leq 1$ cm	6	6	6	9	9	9	6	6	6	6	8	5	6	6	6	
	$m \leq 2$ cm	10	10	12	14	14	14	10	12	16	13	13	10	13	13	13	
	$s \leq 1$ cm	12	8	10	14	14	17	20	16	24	13	24	26	8	8	8	
	$s \leq 2$ cm	19	15	21	31	29	31	37	25	31	27	47	50	17	17	17	
	$\alpha \leq 18^\circ$	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	

* $p < 0.05$

** $p < 0.01$

*** $p < 0.001$

Tables

Table 1. Questionnaire numbers and response rates by age group.

	Distributed questionnaires	Answered questionnaires	Response rate (RR)
Grade 3	128	28	22 %
Grade 5	113	53	47 %
Grade 7	224	150	67 %
Grade 8	205	149	73 %
Adults	82	81	99 %
Σ	752	461	61 %

Figures

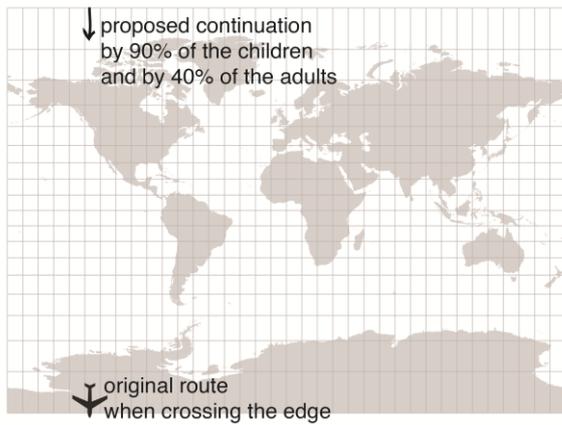


Figure 1. People often mistakenly think that an airplane passing the southern edge of a world map based on a cylindrical projection with a normal aspect continue the route from the northern edge.

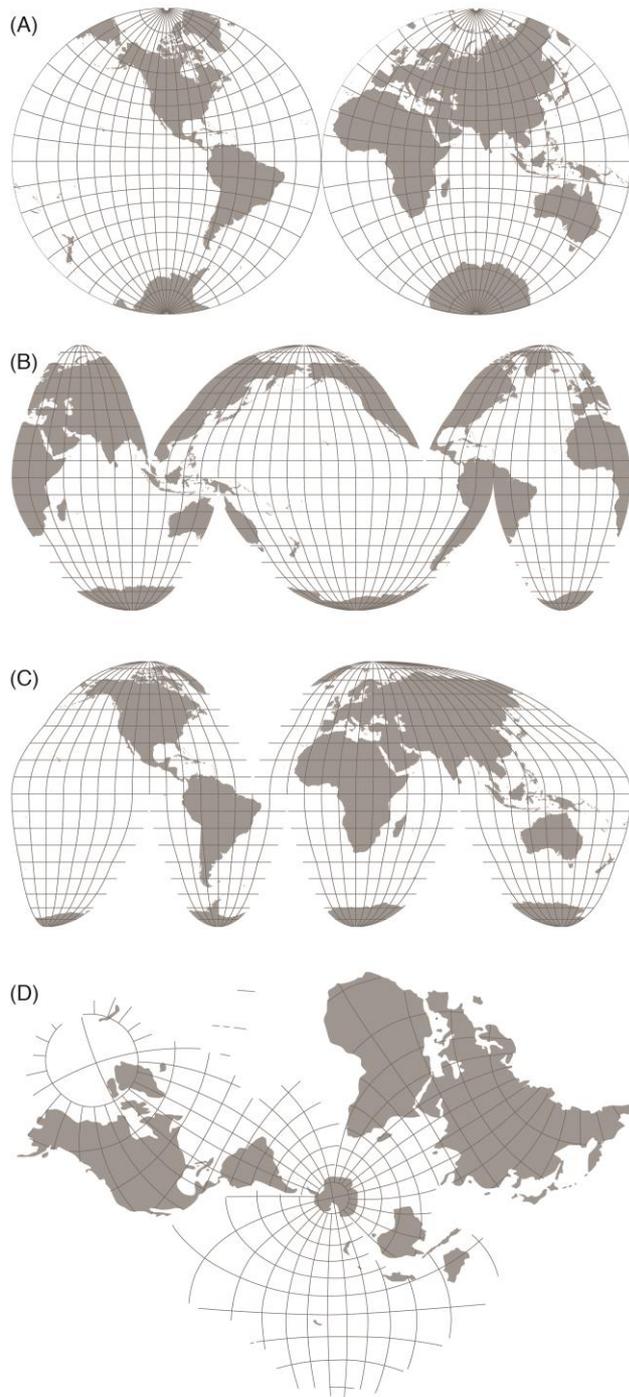


Figure 2. Projections with two or more central meridians. (A) World maps divided into a western and an eastern hemisphere have been used for centuries. (B) Goode's homolosine (ocean). (C) Goode's homolosine (land). (D) Spilhaus interrupted three-lobed August projection.

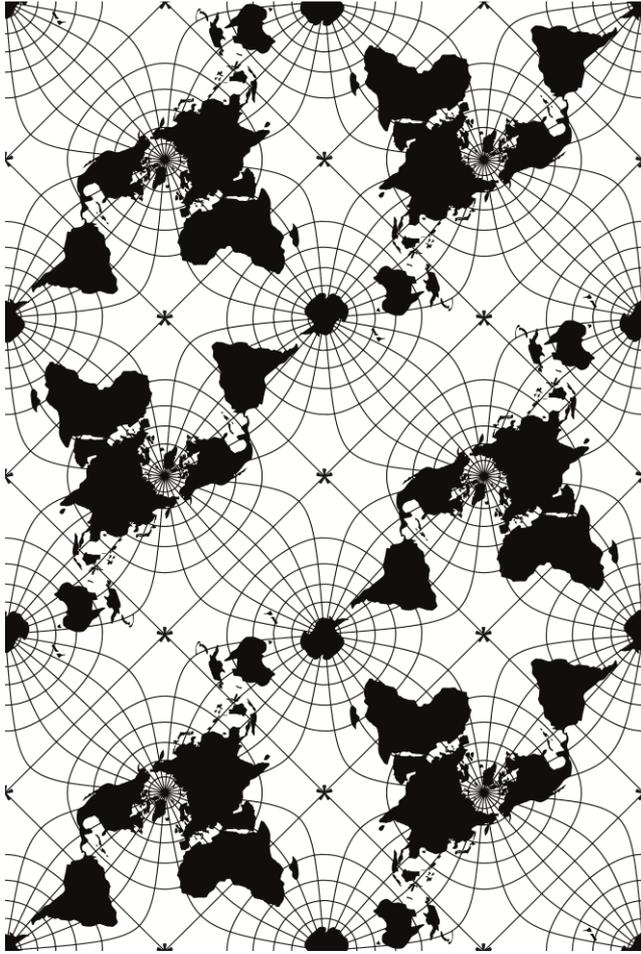


Figure 3. Peirce's quincuncial projection. The points that interrupt the continuations are marked by an asterisk (*).

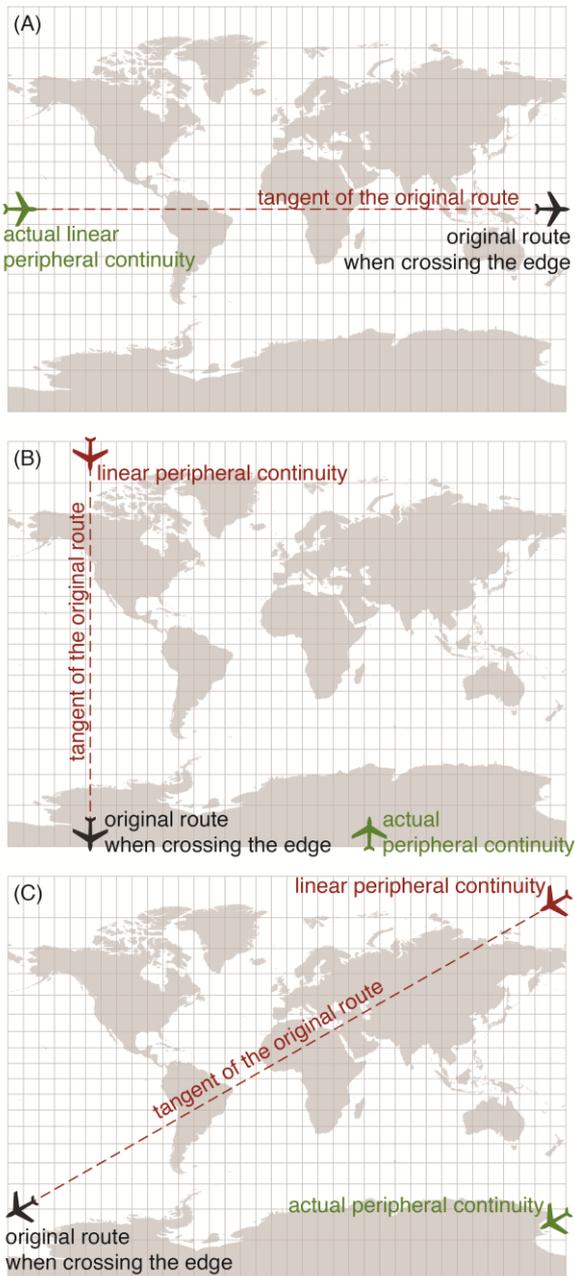


Figure 4. Three examples of linear peripheral continuity and actual peripheral continuity. Map (A) shows an example of actual linear peripheral continuity because the actual location and direction of the reappearance when the edge of the map is crossed is along the tangent of the original route when it crosses the edge. Map (B) and (C) show two different examples of linear peripheral continuity that differ from the actual continuity.

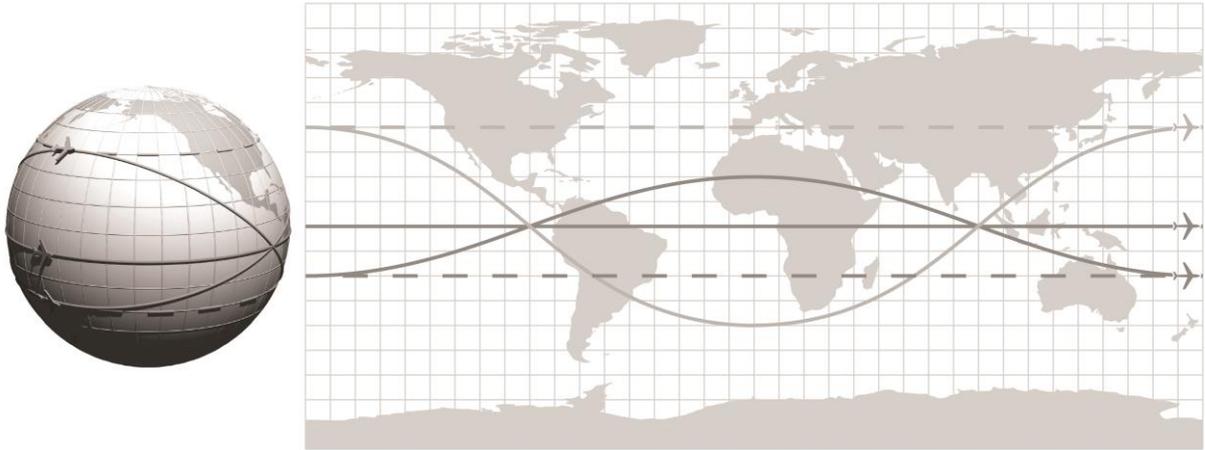
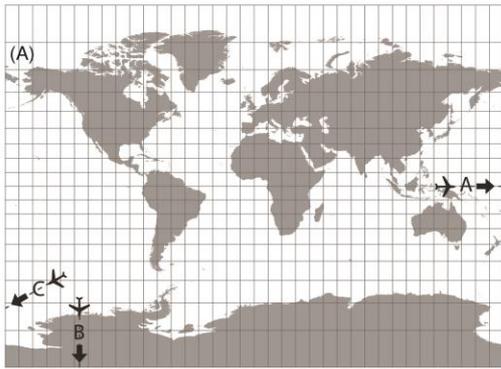
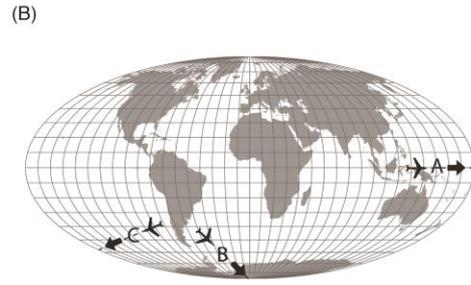


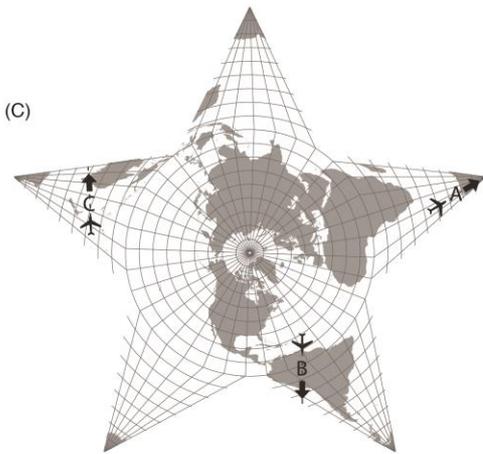
Figure 5. For cylindrical and pseudocylindrical projections, the actual peripheral continuation is linear for all routes crossing the edge along a latitude. However, it is only at the equator that the route around the world actually continues as a straight line on the map. For other latitudes, the great circles (marked as solid lines) return to the tangent of the original route (marked as dashed lines) after a complete lap around the world.



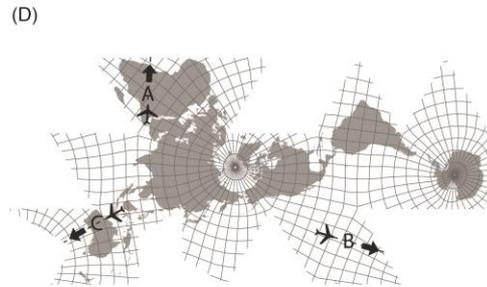
Miller cylindrical projection



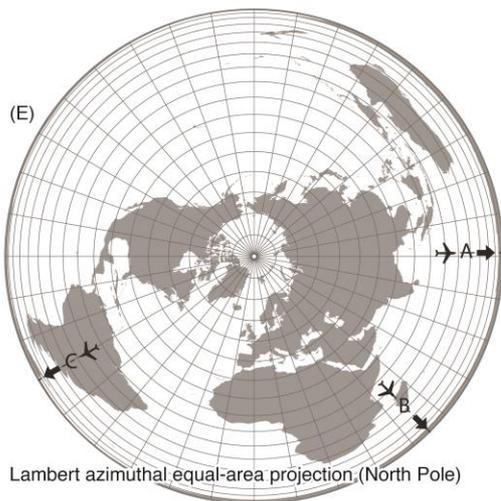
Mollweide projection



Berghaus star projection



Fuller projection



Lambert azimuthal equal-area projection, (North Pole)

Figure 6. World maps based on the five studied map projections. In the questionnaire, the respondents were asked to mark where the airplanes continue after they have crossed the map edge.

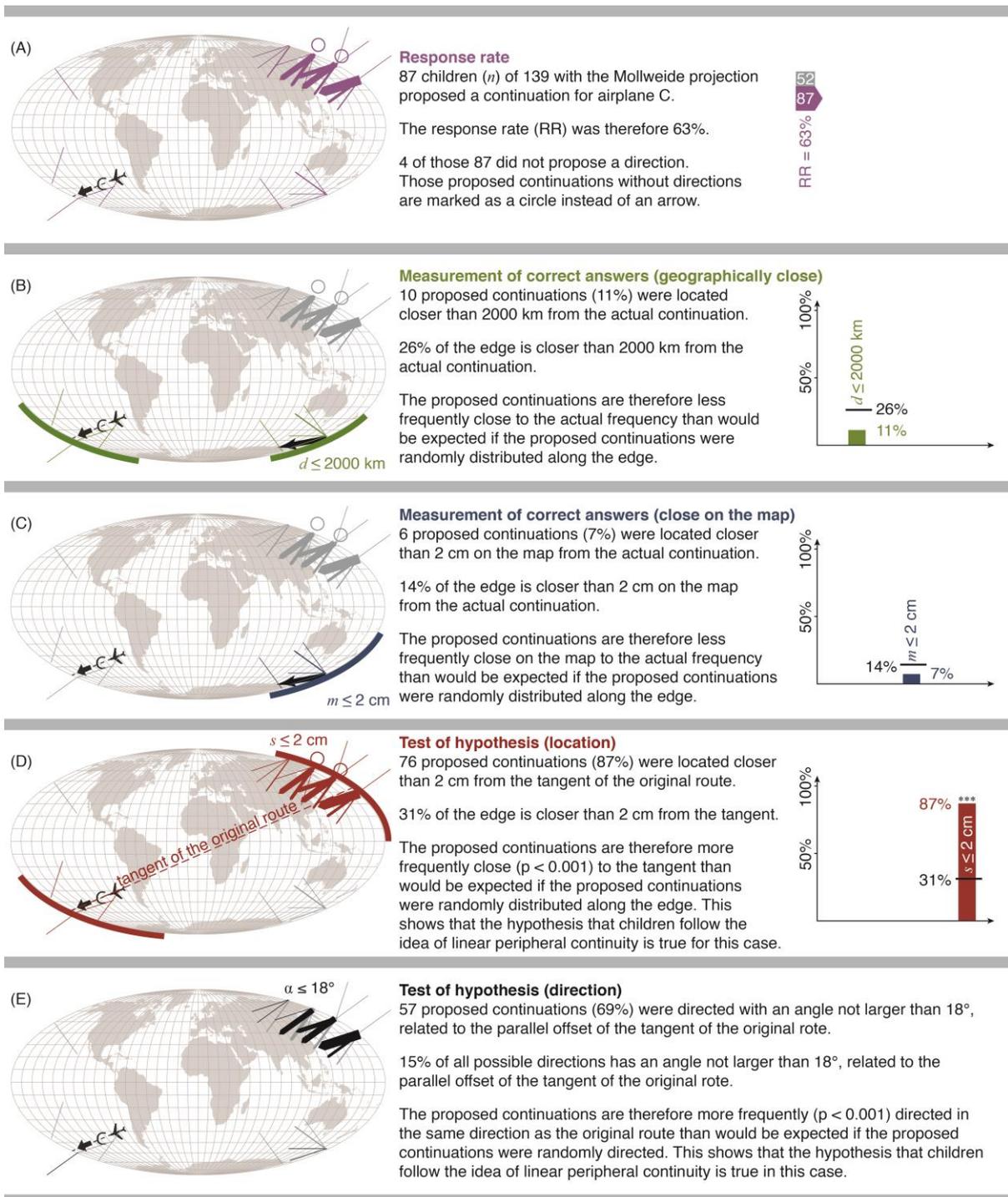


Figure 7. Measurements for the proposed continuations. (A) The number of answers n and the response rate RR. (B) The geographical difference between a proposed and an actual continuation, measured as the distance d . (C) The map difference between a proposed and an actual continuation, measured as the distance m . (D) The degree to which a proposed continuation is linear, measured as the distance s to the tangent of the original route and (E) the angle α on the map that is related to the parallel offset of the tangent.

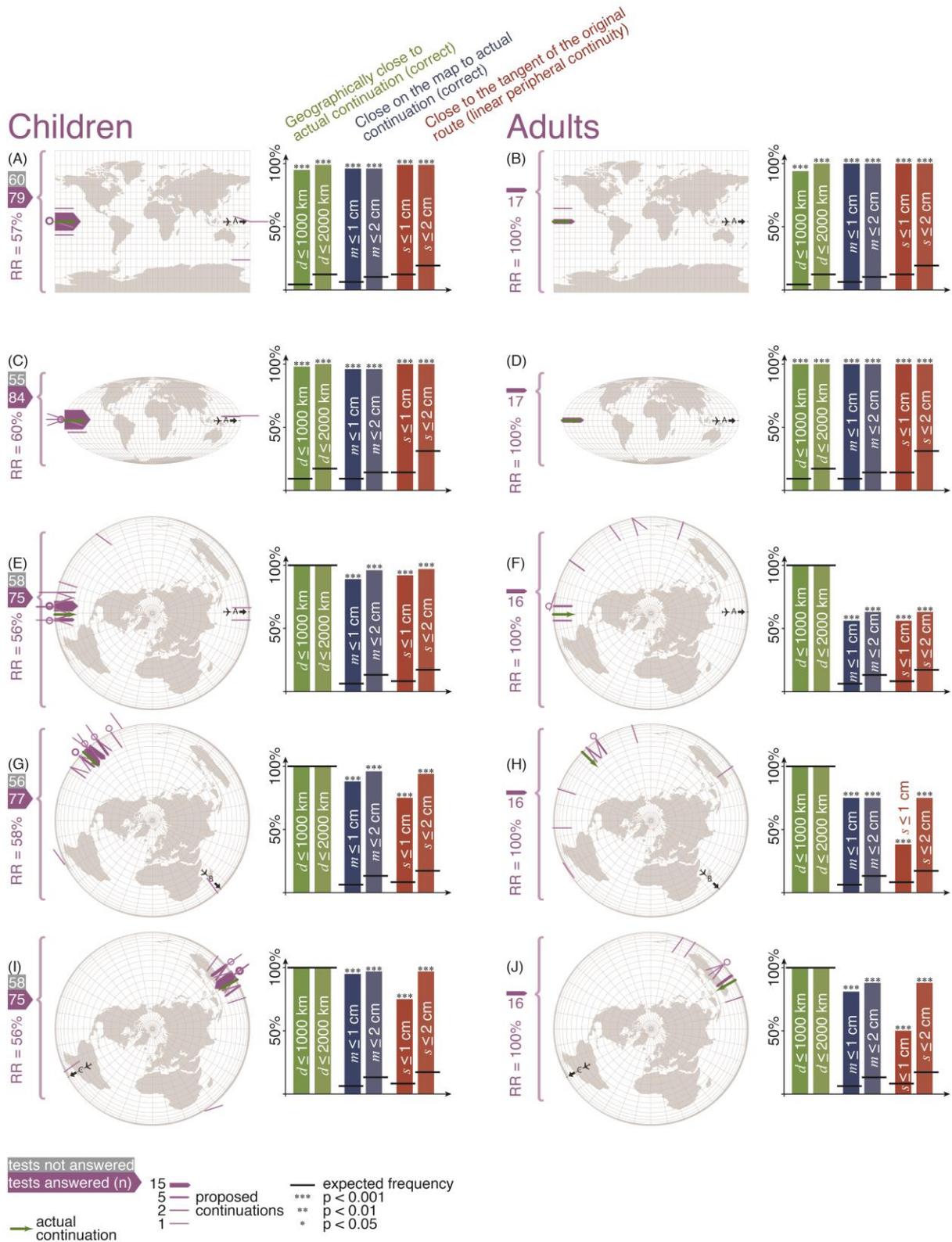


Figure 8. Continuations proposed for the five cases with actual linear peripheral continuation presented as maps and frequencies of d , m , and s in different intervals. The continuations proposed by the children are in the left column, and the continuations proposed by the adults are in the right column (see Figure 7 for an explanation of the measurements).

Children

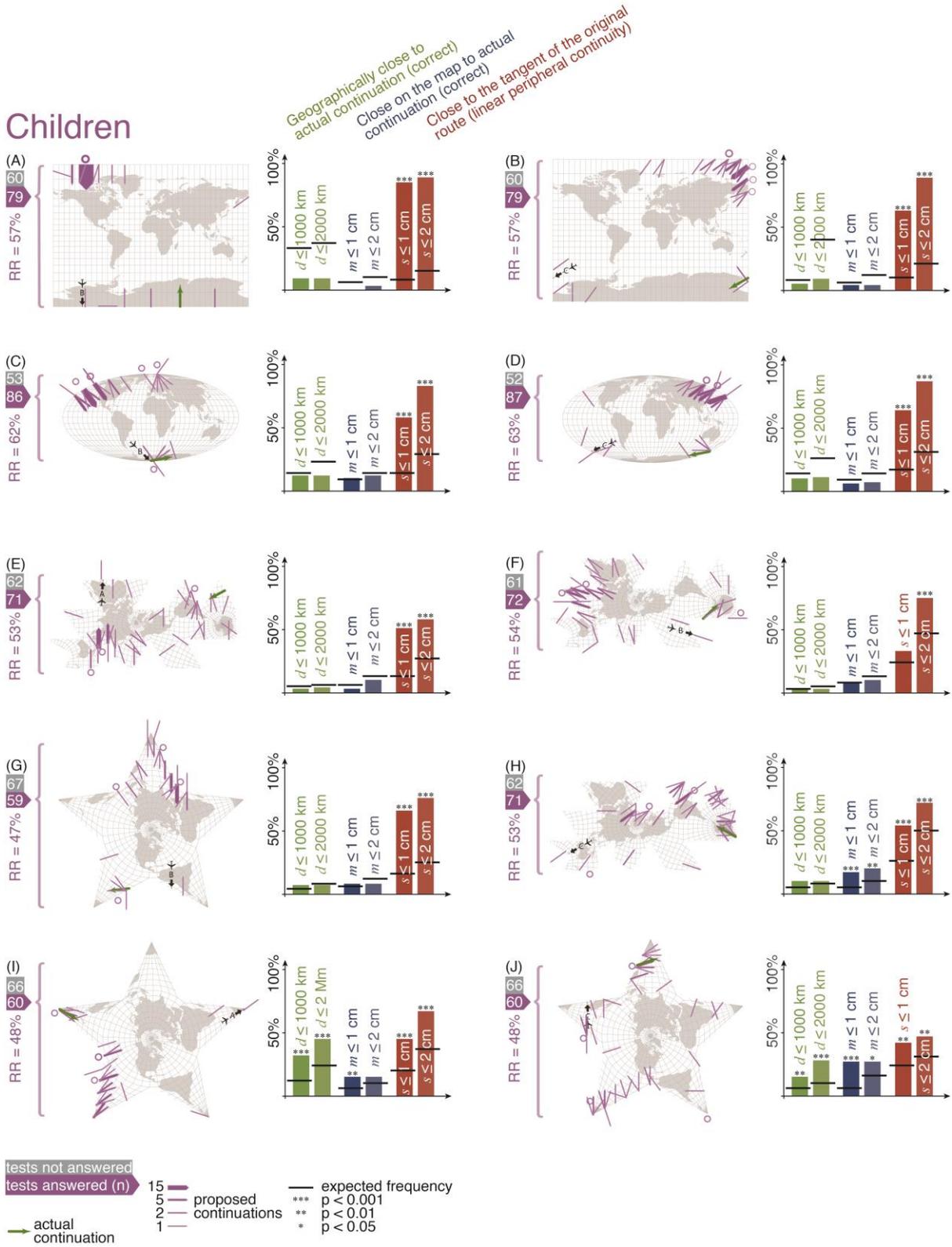
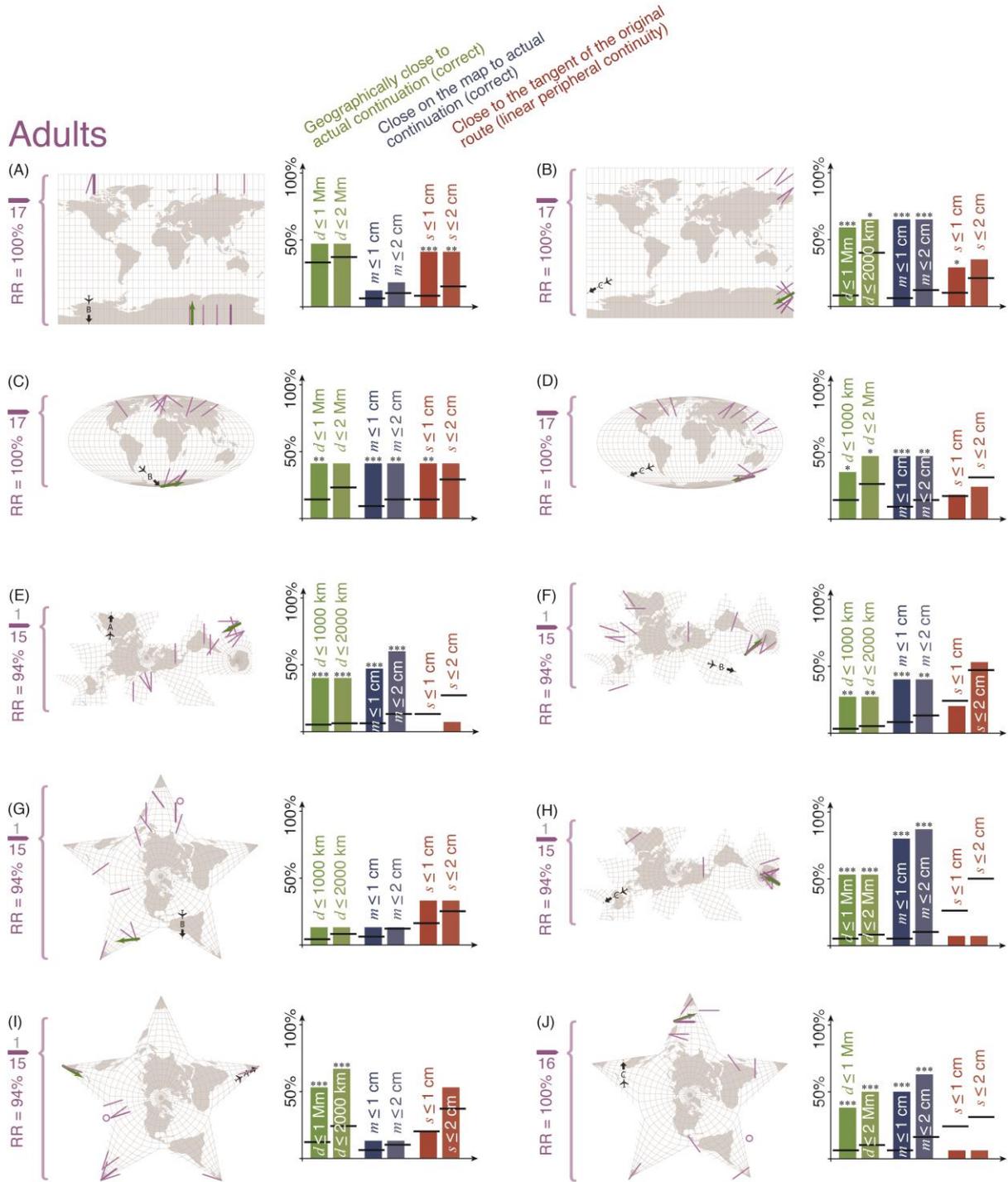


Figure 9. Continuations proposed by the children presented as maps and frequencies of d , m , and s in different intervals. This figure presents the ten cases without actual linear peripheral continuity. In all cases, the responses of the children were significantly ($p < 0.01$) more linear than what would be expected if the answers were randomly distributed along the edge (see Figure 7 for an explanation of the measurements).

Adults



Geographically close to actual continuation (correct)
 Close on the map to actual continuation (correct)
 Close to the tangent of the original route (linear peripheral continuity)

tests not answered
 tests answered (n) 15 — proposed — expected frequency
 5 — continuations *** p < 0.001
 2 — * p < 0.01
 1 — * p < 0.05
 — actual continuation

Figure 10. Continuations proposed by the adults presented as maps and frequencies of d , m , and s in different intervals. This figure presents the ten cases without actual linear peripheral continuity. The adults' responses were significantly ($p < 0.05$) more linear than what would be expected if the answers were randomly distributed along the edge in only the two cases using the cylindrical Miller projection (see Figure 7 for an explanation of the measurements).

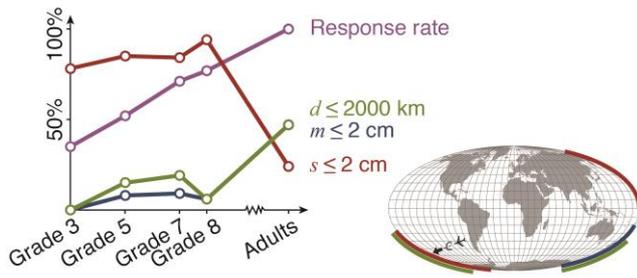


Figure 11. Resulting frequencies of $d \leq 2000$ km, $m \leq 2$ cm, and $s \leq 2$ cm in different age groups for airplane C on the map using the Mollweide projection. The tendency to answer according to linear peripheral continuity (a high frequency of $s \leq 2$ cm) did not decrease with age among the school children but, instead, increased slightly. The map shows the airplane on the map and the parts of the periphery within 2000 km, 2 cm, and 2 cm for the measurements d , m , and s , respectively.