Annihilating branching random walks

Daniel Ahlberg, Stockholm University *August 23, 2022*

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The following is a description of a problem in discrete probability theory.

Consider the particle process on the integer line described as follows: At time zero there is a single particle at the origin. This particle has a Poisson clock which rings at rate $\lambda > 0$. At each ring of the clock the particle sends a copy of itself to each neighbouring integer. Each new particle generated in this way is given its own Poisson clock, with the same rate, independent of remaining clocks, and reproduce according to the same rule. The resulting process is a branching process with an additional spatial component which we will refer to as a *branching random walk* with rate λ.

Consider next the following variation of the above process: At time zero place one particle at each integer point of $\mathbb{Z}$. Colour the particle at the origin *red* and the remaining particles *blue*. Each blue particle evolves as a branching random walk at rate 1. The red particle evolves as a branching random walk at rate λ. Finally, suppose that red and blue particles, once they meet, annihilate on a one for one basis.

One would expect that when $\lambda < 1$, so that red particles reproduce slower than blue, the red particles will not be able to survive in the 'sea of blue' and eventually become annihilated. Does this remain true when $\lambda = 1$ and the two colours reproduce at the same speed? For $\lambda > 1$, so that red particles reproduce faster, is there a positive probability that red survives indefinitely?