Coexistence in a model for competing growth

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The following is a description of a problem in discrete probability theory.

The *square lattice* is the graph whose set of vertices corresponds to all integer pairs $\mathbb{Z}^2$, and where any two points at Euclidean distance 1 are joined by an edge. Consider the following growth model on the square lattice: Each site in $\mathbb{Z}^2$ can be either 'occupied' or 'vacant'. At time zero a single site in $\mathbb{Z}^2$ is occupied. As time evolves, vacant sites with precisely one occupied neighbour becomes occupied at rate 1, and vacant sites with at least two occupied neighbours become occupied at rate $\lambda \geq 1$.

The growth model described above can be extended into a model for competition on the square lattice by introducing two types: At time zero one site is occupied by type 1, another site is occupied by type 2, and remaining sites are vacant. As time evolves, vacant sites become occupied by type 1 or 2 according to the above description, and once a site is occupied by either type, it cannot become occupied by the other.

From the description it is clear that each type may only grow by occupying neighbours. It is therefore possible for one of the types to become surrounded by the other, and hence become 'trapped'. The other possibility is that both types continue to grow indefinitely, and end up occupying infinitely many sides each. In the latter case we say that the two types *coexist*.

For $\lambda = 1$ it is known that coexistence does occur with positive probability. For the model corresponding to $\lambda = \infty$ it is known that coexistence cannot occur. For $1 < \lambda < \infty$ is it possible for the two types to coexist?