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Mathematics Part II

Applied Mathematics: Probability, Differential and Difference Equations, Dynamic Optimization

Probability and Statistics

In the **first part**, we use set theory to define the *sample space*, *outcomes* and *events* and give an axiomatic foundation of probability. We also define *independence*, *conditional probability* and *Bayes theorem*.

We will then study *random variables*, which are functions on the sample space, and their *distribution functions*. We will encounter *discrete* and *continuous* random variables and their attendant *probability mass functions* and *densities*.

Next, we will learn how to summarize and describe distributions of random variables by their *moments*, such as the *mean*, the *variance* and *higher order moments*. We will see how the *moment generating function* can be used to calculate the moments of a distribution.

We will then learn how to find the distribution of a *transformed* random variable.

Finally, we will also briefly review a number of common discrete and continuous probability distributions and the circumstances under which they arise.

In the **second part**, we will prepare for the situation we face in actual data, namely, many random variables, which vary together. We will introduce the *joint* distribution and density functions and how to find *marginal and conditional densities* from them. We also introduce the *covariance* and *correlation*, a measure of how much two random variables move together, as well as the *conditional* expectation and variance.

In the **final part**, we show what can be learned about a distribution (of a random variable) by repeatedly sampling from it. Several *sample statistics*, such as the sample mean and sample variance, and their distributions will be presented. We will then investigate what happens to certain sample statistics and their distributions as the sample size approaches infinity. The convergence concepts we will present here, such as *convergence in probability*, *convergence in distribution*, *the law of large numbers* and *the central limit theorem*, form the basis for inference in much of econometric analysis.

Differential and Difference Equations

In this part of the course we will study change in continuous and in discrete time. In particular, we will learn about ordinary differential and difference equations, both first- and second-order, linear and non-linear. We will learn how to interpret and solve these equations, what their qualitative properties are (existence and uniqueness, stability of solutions) and how to analyze them using graphical methods when closed-form solutions are not possible. Finally, we will use our knowledge from linear algebra to solve systems of differential and difference equations.

Dynamic Optimization

TBA

Readings

Basic Integration

Essential Mathematics for Economic Analysis, Sydsaeter, Hammond, Strom, and Carvajal (2016), Chapter 9
Further Mathematics for Economic Analysis, Sydsaeter, Hammond, Seierstad and Strom (2008), Chapter 4

Probability

*G. Grimmett and D. Stirzaker, Probability and Random Processes
G. Casella and R.L. Berger, Statistical Inference

Differential and Difference Equations

Further Mathematics for Economic Analysis, Sydsaeter, Hammond, Seierstad and Strom, Chapters 5,6 and 11
Additional material from MIT 18.03 OCW, which will be made available on Athena.
Good to read as review/prerequisite: Chapters in Simon and Blume on complex numbers, trigonometric functions and Integral Calculus A2, A3, A4.